CANADA

DOMINION BUREAU OF STATISTICS

HEALTH AND WELFARE DIVISION

Health and Welfare Technical Report No. 1

CANADIAN ABRIDGED LIFE TABLES, 1961-1963

by

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OTTAWA

May, 1966

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1. Introduction

Canadian Abridged Life Tables, 1961-1963 is not to be regarded as an official Dominion Bureau of Statistics publication, but merely as a report of a project carried out by the Analysis Unit of the Vital Statistics Section. Abridged life tables, although less accurate than complete life tables, have the advantage of being rapidly constructed by using available computer techniques. These abridged life tables have been constructed by using the MATOP 6 program prepared for the IBM 360 computer [4]. The assumptions, method, and data for the construction of these abridged life tables are presented below. The authors of the report are grateful to Dr. Y. Kasahara for her professional advice.

2. Data Used

These abridged life tables are based on the following data: June 1, 1962 estimates of the population of Canada by age group and sex; reported deaths in Canada for the three years 1961, 1962, and 1963, infant deaths being required by year of birth; and reported births in Canada for the four years 1960, 1961, 1962, and 1963. These data are recorded in the Appendix of the report.

3. Calculation of the Death Rate ${}_nm_x$

The death rate ${}_nm_x$ is a three-year average of deaths divided by the June 1, 1962 population estimate, for each of the age groups. Thus, for the age group x to $x+n-1$,

$${}_nm_x = \frac{D_x^{z-1} + D_x^z + D_x^{z+1}}{3P_x^z}, \quad (1)$$

where z is the calendar year,

D_x^z is the number of deaths during the calendar year z in the age group x to x+n-1,

P_x^z is the population exposed to the risk of death during the calendar year z in the age group x to x+n-1, and

n is the number of years during which an individual remains in one age group, usually five in our case.

4. Relation between the Death Rate ${}_nm_x$ and the Probability of Dying ${}_nq_x$

The relationship between the death rate and the probability of dying depends on the grouping of the basic data. We have chosen mainly five-year age groups for both the deaths and the population. The method of calculating the probability of death during the first year of life is described below.

Defined by T.N.E. Greville [5] this relation takes into account the non-linearity of l_x, and is

$$ {}_nq_x = \frac{{}_nm_x}{\frac{1}{n} + {}_nm_x \left[\frac{1}{2} + \frac{n}{12}\left({}_nm_x - \log_e c\right)\right]} . \quad (2) $$

Here, ${}_nq_x$ is the probability of dying within n years after attaining age x. Also, the constant c comes from the assumption that ${}_nm_x$ follows Gompertz' law

$$ {}_nm_x = Bc^x . \quad (3) $$

In his original paper, B. Gompertz restricted the use of the formula to the period from about age 10 or 15 to age 55 or 60, and it is not possible to use the formula over the entire range of ages without making a change in the constants at some point [2] . At the younger ages, however, ${}_nq_x$ is not affected much by variations in c. At the higher ages, mortality rates conform closely to Gompertz' law in most experiences. For these reasons, c may be assumed as a constant for the entire range of the life table. Values of $\log_e c$ lie largely between 0.08 and 0.104. In any case, c may be approximated from an average, at the higher ages, of

$$c = \left[\frac{{}_n m_{x+n}}{{}_n m_x}\right]^{\frac{1}{n}} \qquad (4)$$

For these abridged life tables, $\log_e c$ was approximated by averaging its values for the ten age groups 10-14 to 55-59 inclusive, and is recorded in each case.

5. Calculation of T_x and $\mathring{e}_x$

The formulas which allow one to determine the expectation of life are as follows:

l_0 = 100,000 is the number chosen as the radix in all life tables;

$l_1 = l_0 - d_0$, where d_0 is the number of deaths at age 0 out of 100,000 live births;

${}_n d_x = l_x \cdot {}_n q_x$, where ${}_n d_x$ is the number of persons dying in the age interval x to x+n-1 out of 100,000 live births;

$l_x - {}_n d_x = l_{x+n}$, where l_{x+n} is the number of persons, starting with a cohort of 100,000 live births, who survive to the exact age x+n marking the beginning of the age interval;

${}_n L_x \doteq \dfrac{{}_n d_x}{{}_n m_x}$, where ${}_n L_x$ denotes the number of persons in the hypothetical stationary population in the age interval x to x+n-1;

$T_x = T_{x+n} + {}_n L_x$, where T_x denotes the total number of persons in the hypothetical stationary population in the age interval x to x+n-1 and all subsequent age intervals;

$\mathring{e}_x = \dfrac{T_x}{l_x}$, where $\mathring{e}_x$ is the average remaining lifetime at age x, also called the expectation of life.

6. The First Year of Life

The value of $_1q_0$, the proportion of liveborn infants dying before reaching age 1, is computed from birth and death statistics, being taken as equal to the adjusted infant death rate [3]. The adjustment is made by allocating the deaths of infants occurring during a given year to the year in which the infants were born. The infant deaths so allocated are then related to the births occurring in the respective year of birth. The expression for computing the adjusted infant mortality rate may be written

$$_1q_0 = \frac{D(1-f)}{E} + \frac{Df}{E'}, \qquad (5)$$

where D is the number of infant deaths occurring the given period z-1 to z+1,

f, referred to as the "separation factor," is the ratio of deaths during the given period z-1 to z+1 among infants born in a preceding year to the total infant deaths of the given period z-1 to z+1,

E is the number of births occurring in the given period z-1 to z+1, and

E' is the number of births occurring in the period z-2 to z.

The stationary population in the first year of life was obtained by the formula.

$$L_0 = l_0 - (1-f)\, d_0. \qquad (6)$$

7. Further Remarks

Seven significant figures were retained by the computer throughout the entire calculations. In the published tables, all the l_x values have been rounded to the nearest integer, while the published d_x values have been obtained by differencing the published l_x column and, for that reason, differ slightly in some cases from the figures which would result from rounding the d_x values as originally calculated. Similarly, the published values of T_x have all been rounded to the nearest integer and the published L_x values were obtained by differencing the published T_x column, and therefore differ slightly in some cases from the figures which would result from rounding the originally calculated L_x values directly.

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Table 1. Abridged Life Table for Male Population: Canada, 1961-1963

$\log_e c = 0.075778$

Age Group	$_nm_x$	$_nq_x$	l_x	$_nd_x$	$_nL_x$	T_x	$\mathring{e}_x$
Under 1		0.030210	100,000	3,021	97,380	6,840,420	68.40
1- 4	0.001228	0.004899	96,979	475	387,013	6,743,040	69.53
5- 9	0.000644	0.003218	96,504	311	481,791	6,356,027	65.86
10-14	0.000544	0.002716	96,193	261	480,355	5,874,236	61.07
15-19	0.001125	0.005609	95,932	538	478,398	5,393,881	56.23
20-24	0.001733	0.008631	95,394	823	475,039	4,915,483	51.53
25-29	0.001519	0.007566	94,571	716	471,175	4,440,444	46.95
30-34	0.001621	0.008075	93,855	758	467,498	3,969,269	42.29
35-39	0.002263	0.011257	93,097	1,048	463,027	3,501,771	37.61
40-44	0.003424	0.016983	92,049	1,563	456,574	3,038,744	33.01
45-49	0.005707	0.028156	90,486	2,548	446,433	2,582,170	28.54
50-54	0.009490	0.046408	87,938	4,081	430,052	2,135,737	24.29
55-59	0.015331	0.073964	83,857	6,202	404,561	1,705,685	20.34
60-64	0.024043	0.113676	77,655	8,828	367,157	1,301,124	16.76
65-69	0.035585	0.163838	68,827	11,276	316,890	933,967	13.57
70-74	0.053978	0.238316	57,551	13,715	254,089	617,077	10.72
75-79	0.081579	0.338520	43,836	14,840	181,900	362,988	8.28
80-84	0.124745	0.470883	28,996	13,654	109,454	181,088	6.25
85-89	0.186655	0.618164	15,342	9,484	50,811	71,634	4.67
90+	0.281341	1.000000	5,858	5,858	20,823	20,823	3.55

Table 2. Abridged Life Table for Female Population: Canada, 1961-1963

$\log_e c = 0.076105$

Age Group	${}_nm_x$	${}_nq_x$	l_x	${}_nd_x$	${}_nL_x$	T_x	$\mathring{e}_x$
Under 1		0.023619	100,000	2,362	97,986	7,431,745	74.32
1- 4	0.000965	0.003853	97,638	376	389,838	7,333,759	75.11
5- 9	0.000422	0.002109	97,262	205	485,829	6,943,921	71.39
10-14	0.000288	0.001438	97,057	140	484,957	6,458,092	66.54
15-19	0.000513	0.002560	96,917	248	484,005	5,973,135	61.63
20-24	0.000597	0.002982	96,669	288	482,671	5,489,130	56.78
25-29	0.000702	0.003506	96,381	338	481,112	5,006,459	51.94
30-34	0.000899	0.004488	96,043	431	479,205	4,525,347	47.12
35-39	0.001346	0.006709	95,612	642	476,556	4,046,142	42.32
40-44	0.002059	0.010243	94,970	972	472,570	3,569,586	37.59
45-49	0.003312	0.016434	93,998	1,545	466,361	3,097,016	32.95
50-54	0.005172	0.025548	92,453	2,362	456,708	2,630,655	28.45
55-59	0.008023	0.039370	90,091	3,547	442,090	2,173,947	24.13
60-64	0.012934	0.062749	86,544	5,431	419,859	1,731,857	20.01
65-69	0.021066	0.100292	81,113	8,135	386,162	1,311,998	16.17
70-74	0.034112	0.157590	72,978	11,500	337,146	925,836	12.69
75-79	0.058331	0.255018	61,478	15,678	268,775	588,690	9.58
80-84	0.101560	0.403244	45,800	18,469	181,849	319,915	6.99
85-89	0.165833	0.573586	27,331	15,677	94,533	138,066	5.05
90+	0.267718	1.000000	11,654	11,654	43,533	43,533	3.74